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Optimal Multilevel Control for Large Scale Interconnected Systems

Ahmed M. A. Alomar, Bakr E. M. Shamseldin
Electrical Networks Department, Public Authority for Applied Education and Training ,KUWAIT

Abstract .
A mathematical model of the finishing mill as an example of a large scale interconnected dynamical system is represented. First the system response due to disturbance only is presented. Then,the control technique applied to the finishing hot rolling steel mill is the optimal multilevel control using state feedback. An optimal controller is developed based on the integrated system model, but due to the complexity of the controllers and tremendous computational efforts involved, a multilevel technique is used in designing and implementing the controllers .The basis of the multilevel technique is described and a computational algorithm is discussed for the control of the finishing mill system . To reduce the mass storage , memory requirements and the computational time of the processor, a sub-optimal multilevel technique is applied to design the controllers of the finishing mill . Comparison between these controllers and conclusion is presented.

I. INTRODUCTION

The first step in analysis , design , and synthesis of real-life problems is the development of a "mathematical model" which can be a substitute of the real system .In any modeling two tasks often conflicting factors prevail "simplicity" and "accuracy" . On one hand , if a system model is oversimplified presumably for computational effectiveness , incorrect conclusions may be drawn from it in representing the actual system . On the other hand a highly detailed model would lead to a great deal of unnecessary complications and should a feasible solution be attainable, the extent of resulting details may become so vast that further investigations on the system behavior would become impossible with questionable practical values [1] [2]. Then it is clear that a mechanism by which a compromise can be made between a complex , more accurate model and a simple , less accurate model , is needed.

First, a layout of the strip mill with the function of each part is given . Then , from the steady state equations of the mill , a mathematical model is derived for the three finishing mill stands . In order to apply modern control theory to the mill , a state space model is obtained . A simplified state space model is obtained by approximating the pure delay and reducing the order of the system[6] , [7] .

FUNCTIONAL DESCRIPTION OF HOT ROLLING STEEL MILL

The layout of the hot rolling mill under consideration is shown in Figure 1 . The steel slabs are heated in furnace (1) then transferred to the delivery roll table (2). The slab passes the roughing descaler (3) to remove scale from its surface . After wards , the slab enters the roughing mill (4) , which consists of horizontal and vertical rolls, where its thickness is reduced . After this stage, it passes the cropping shear (5) to cut its front and tail ends. Then, it enters the finishing mill train (6) and then passes over the run-out roller table (7) . Finally, it enters the coiling area (8)

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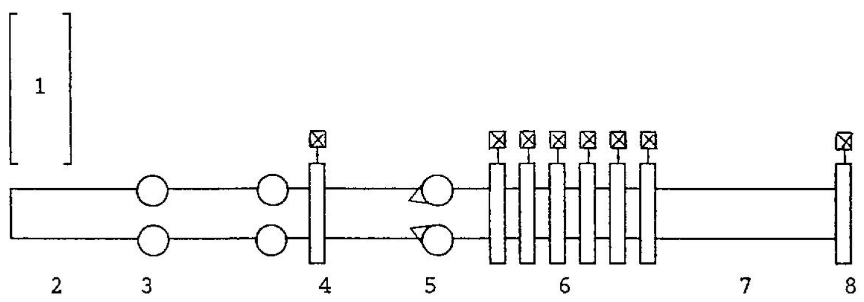

Figure 1 Lay-out of hot strip mill.

The main objectives of the automation of steel mills are to increase efficiency, productivity, and product quality. As regard to the size of production in steel , it is clear that an increase in production or an improvement in the output of only a few percent may represent a very large increase in profit [3] . The finishing mill is the last stage where small reduction in the slab thickness is carried out . This yields that the accuracy of the gauges in the finishing stands is directly related to the quality of the products.

II. STATE SPACE MODEL OF THE FINISHING MILL

$$\dot{X}=AX+BU+ED \qquad X(0)=0 \qquad (1)$$

Where :

X: is 6-dimensional state vector.

U: is 3- dimensional control vector .

D: is 2-dimensional disturbance functions .

$\underline{X}= [\, W_1 \quad W_2 \quad W_3 \quad T_1 \quad T_2 \quad T_3 \,]^t$

$\underline{U}= [\, e_{a1} \quad e_{a2} \quad e_{a3} \,]^t$

$\underline{D}= [\, d_1 \quad d_2 \,]^t$

W_1 W_2 W_3 : stands for the angular velocities deviations from operating values of the three finishing mill stands .

T_1 T_2 T_3 :stands for the outlet tension deviations from the operating values of the three finishing mill stands .

e_{a1} e_{a2} e_{a3} : stands for the armature voltage deviations from the operating values of the motors controlling the three finishing mill stands .

d_1 : deviation of slab temperature from operating value at inlet of mill no. 1

d_2 :slab thickness at inlet of mill no. 1

Matrices A,B and E are given as follows :

$$A=\begin{bmatrix} -430 & 0 & 0 & 4.8 & 0 & 0 \\ 0 & -176 & 0 & -3 & 2.52 & 0 \\ 0 & 0 & -184 & -0.91 & -4.052 & 2.5 \\ 0 & 375 & 0 & -97.5 & -3.8 & 0 \\ 0 & -630 & 410 & -103.7 & -125.6 & -11 \\ 0 & 0 & -630 & 200 & 102.8 & -31.1 \end{bmatrix}$$

$$B=\begin{bmatrix} 3.9 & 0 & 0 \\ 0 & 2.5 & 0 \\ 0 & 0 & 3.7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad E=\begin{bmatrix} 0.107 & -3.6 \\ 0.026 & -0.73 \\ -0.0136 & 0.38 \\ 2.44 & -43.6 \\ -2.37 & 65.2 \\ 3.0 & -82.0 \end{bmatrix}$$

System response due to the disturbance only (no control):

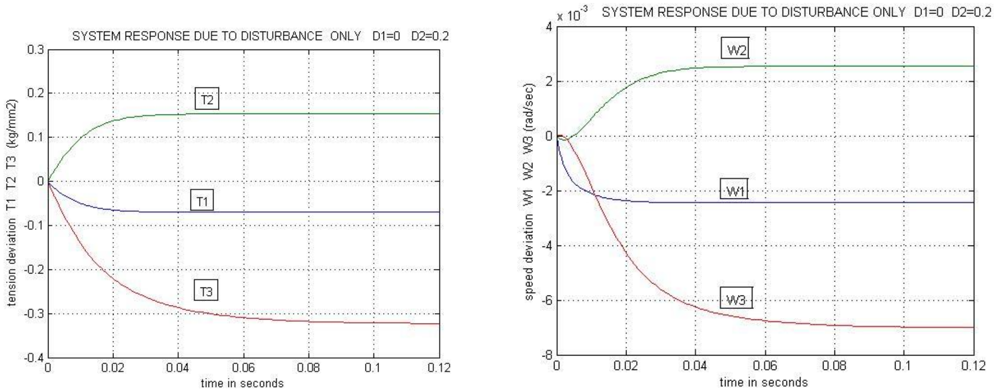

Figure 2 Finishing mill response due to disturbance d1=0 d2=0.2

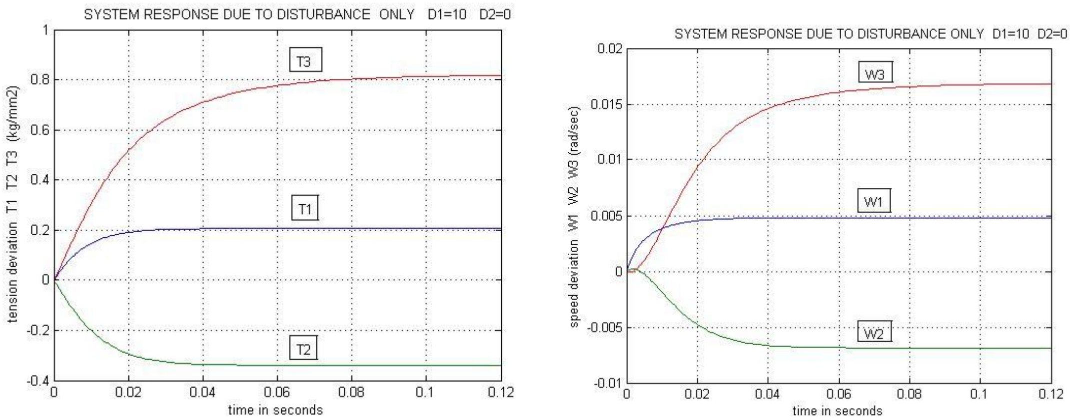

Figure 3 Finishing mill response due to disturbance d1=10 d2=0

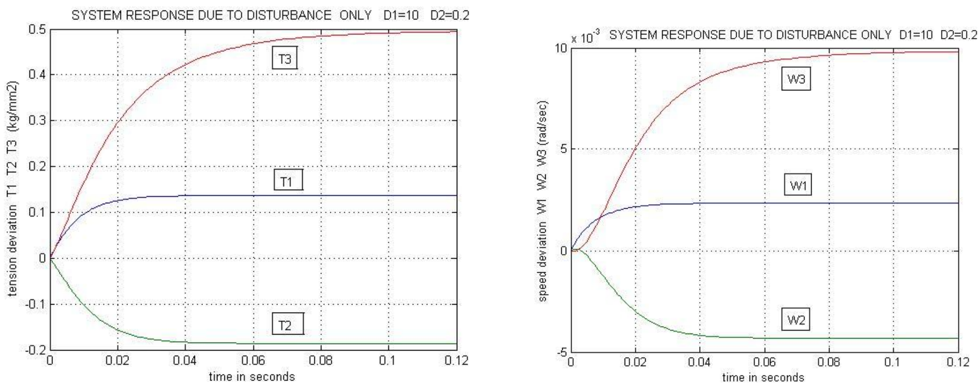

Figure 4 Finishing mill response due to disturbance d1=10 d2=0.2

III. OPTIMAL MULTILEVEL CONTROL

The optimal control law takes the form of a state feedback control , where the state X contains all pertinent information about the system . There are instances for which the state variables are all measurable, i.e. , outputs. If it is not the case, then there are effective means available for estimating or reconstructing the state variables (observers) from the available inputs and outputs.

The necessity of computer solutions has limited real time implementation of optimal control theory in the past Still , off-line optimal solutions have provided valuable standards of comparison for evaluating easier-toimplement suboptimal control schemes . Off-line solution of the Riccati equation allows optimal feedback control matrices to be precomputed . Often , the constant steady state version of the feedback matrices gives sufficiently good control [6].

3.1 OPTIMAL CONTROL DESIGN

The system is represented in the state form as in equation (1):

$$\dot{X}=AX+BU+ED \qquad X(0)=0$$

Then , for optimal performance the following quadratic criteria is introduced

$$J=1/2 \int_0^T [< X(t), Q\, X(t) > + < U(t), R\ \ U(t) >]dt \tag{2}$$

Where , < > is the scalar product

$$<X(t) , Q\, X(t) > = X^t(t)\, Q\, X(t)$$

$$<U(t) , R\, U(t) > = U^t (t)\, R\, U(t)$$

R= is a positive definite weighting matrix for a control.

Q= is a positive semi definite weighting matrix for the states divided into two parts.

Q1: weight for the speed states.

Q2: weight for the outlet tension states.

Then the Riccati equation:

$$\dot{K} = -KA - A^t K + KBR^{-1} B^t K - Q \qquad K(T) = 0 \tag{3-a}$$

$$\dot{h} = (KBR^{-1}B^t - A^t)h + KED \qquad h(T) = 0 \tag{3-b}$$

Then the optimal control law

$$U^* (t) = - R^{-1} B^t\, K\, X(t) + R^{-1} B^t\, h(t) \tag{4}$$

where K and h are the feedback and feed forward gains respectively obtained as the solution of Riccati equations (3).

Optimal controllers for applications of such large interconnected dynamic systems are complex and require computers with large storage capacity and large computational time , is needed , for solving such systems . In the finishing mill system ,even though , the model order used in this paper is the reduced one but we still have :

36 differential equations for the feedback gains k .
6 differential equations for the feed forward gains h .
in addition to the system differential equations .

Another alternative control for such systems is the well known multilevel technique control which will be applied to control the finishing mill system .

IV. OPTIMAL MULTILEVEL CONTROL DESIGN [8]

This technique is used to simplify the design of the controllers for large interconnected dynamical systems . First, decompose the system into subsystems with minimum coupling between these subsystems .Then , **local optimal controllers** are derived for each subsystem assuming that all the coupling parameters are known (**first level control**) . A second level controller is designed to satisfy the goals of the original system , and update the coupling variables making use of the information about each subsystem states . The fundamental concept of this scheme is to design local controllers to optimize each subsystem ignoring coupling between subsystems . Then **global optimal controller** will be used to minimize the effect of coupling and improve the overall system performance (**second level control**). The finishing mill model (1) should be modified by rearranging the states X so that each stand will be considered as a subsystem, with states (W,T) , and control e_a making the necessary modifications in the A,B and E matrices as follows:

$$\dot{X}=AX+BU+ED \qquad X(0)=0 \tag{5}$$

$X=[\,W_1 \;\; T_1 : W_2 \;\; T_2 : W_3 \;\; T_3]^t$

$U=[\,e_{a1} \; : \; e_{a2} \; : \; e_{a3}]^t$

$$A=\left[\begin{array}{cc:cc:cc} -430 & 4.8 & 0 & 0 & 0 & 0 \\ -640 & -97.5 & 375 & -3.8 & 0 & 0 \\ \hdashline 0 & -3 & -176 & 2.52 & 0 & 0 \\ 0 & -103.7 & -630 & -125.6 & 410 & -11 \\ \hdashline 0 & -.91 & 0 & 0 & -4.052 & -184 \\ 0 & 200 & 0 & 102.8 & -630 & -31.1 \end{array}\right]$$

$$B=\left[\begin{array}{c:c:c} 3.9 & 0 & 0 \\ 0 & 0 & 0 \\ \hdashline 0 & 2.5 & 0 \\ 0 & 0 & 0 \\ \hdashline 0 & 0 & 3.7 \\ 0 & 0 & 0 \end{array}\right] \qquad E=\left[\begin{array}{cc} 0.107 & -3.6 \\ 2.44 & -43.6 \\ \hdashline 0.026 & -0.73 \\ -2.37 & 65.2 \\ \hdashline -0.0136 & 0.38 \\ 3.0 & -82.0 \end{array}\right]$$

4.1 DESIGN OF OPTIMAL LOCAL CONTROLLERS (First Level control)

Equation (5) is composed of three linear subsystems with interconnections and can be described by the equation

$$\dot{X}_i=A_i X_i+ B_i U_i^L+ E_i D_i + C_t(t , x) \qquad X(0)=0 \tag{6}$$

Then , by ignoring the interconnections , each subsystem is described as

$$\dot{X}_i=A_i X_i +B_i U_i^L+E_i D_i \qquad i = 1 ,2 ,3 \tag{7}$$

For optimality, use the cost function

$$J_i= 1/2 \int_0^T [< Xi, Qi, X > + < UiL,\; Ri \;\; Ui(t)L >]dt \tag{8}$$

And the optimal local controller :

$$U^{*}{}_i{}^L(t) = - R_i^{-1} B_i^{t} K_i \; X_i (t) + R_i^{-1} B_i^{t} \; h_i (t) \tag{9}$$

where K_i and h_i are the solutions for the Riccati equation for the i^{th} subsystem, namely

$$\dot{K}_i = -K_i A_i – A_i^t K_i+ K_i B_i R_i^{-1} B_i t K_i – Q_i \qquad K_i (T) = 0$$

$$\dot{h}_i= (K_i \;\; B_i \;\; R_i^{-1} \;\; B_i^t \; – A_i^t) h_i + K_i E_i D_i \qquad h_i (T) = 0$$

Each closed-loop subsystem is represented for the i[th] subsystem as :

$$\dot{X}_i = (A_i - B_i R_i^{-1} B_i^t K_i)X_i + B_i R_i^{-1} B_i^t h_i + E_i D_i \tag{10}$$

The effect of the coupling between different subsystems will be considered in the second control level . As a result of decomposing the system into three subsystems , each subsystem will have an interconnection matrix C_i which represents the coupling between the i^{th} subsystem and other subsystems affecting it . Then each subsystem after simplification is expressed as follows :

The First Subsystem

$$\dot{W}_1 = -430\, W_1 + 4.8\, T_1 + 3.9\, e_{a1} + 0.107\, d_1 - 3.6\, d_2 \tag{11-a}$$

$$\dot{T}_1 = -640\, W_1 - 97.5\, T_1 + 375\, W_2 - 3.8\, T_2 + 2.44\, d_1 - 43.6\, d_2 \tag{11-b}$$

The Second Subsystem

$$\dot{W}_2 = -176\, W_2 + 2.52\, T_2 + 3.0 T_1 + 2.5\, e_{a2} + 0.026\, d_1 - 0.73\, d_2 \tag{12-a}$$

$$\dot{T}_2 = -\, 630\, W_2 - 125.6\, T_2 + 410\, W_3 - 11\, T_3 + 103.7 T_1 - 2.37\, d_1 + 65.2\, d_2 \tag{12-b}$$

The Third Subsystem

$$\dot{W}_3 = -184\, W_3 + 2.5\, T_3 - 0.91 T_1 - 4.052 T2 + 3.7\, e_{a3} - 0.0136\, d_1 + 0.38\, d_2 \tag{13-a}$$

$$\dot{T}_3 = -\, 630\, W_3 - 31.1\, T_3 + 200 T_1 + 102.8 T_2 + 3\, d_1 - 82\, d_2 \tag{13-b}$$

4.2 DESIGN OF GLOBAL CONTROLLER (Second Level Control)

The integrated system corresponding to (7) is represented as follows

$$\dot{X}=A_dX+B_dU_T+ED+CX \qquad X(0)=0 \tag{14}$$

$$A_d=\mathrm{diag}[\ A_1 \quad A_2 \quad A_3\ ]$$

$$B_d=\mathrm{diag}[\ B_1 \quad B_2 \quad B_3\ ]$$

$$A_d=\begin{bmatrix} -430 & 4.8 & 0 & 0 & 0 & 0 \\ -640 & -97.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & -176 & 2.52 & 0 & 0 \\ 0 & 0 & -630 & -125.6 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4.052 & -184 \\ 0 & 0 & 0 & 0 & -630 & -31.1 \end{bmatrix}$$

$$B_d=\begin{bmatrix} 3.9 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2.5 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3.7 \\ 0 & 0 & 0 \end{bmatrix} \qquad E=\begin{bmatrix} 0.107 & -3.6 \\ 2.44 & -43.6 \\ 0.026 & -0.73 \\ -2.37 & 65.2 \\ -0.0136 & 0.38 \\ 3.0 & -82.0 \end{bmatrix}$$

$$C=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 375 & -3.8 & 0 & 0 \\ 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & -103.7 & 0 & 0 & 410 & -11 \\ 0 & -0.91 & 0 & -4.052 & 0 & 0 \\ 0 & 200 & 0 & 102.8 & 0 & 0 \end{bmatrix}$$

where all interconnections between the three subsystems are extracted from matrix A and added in the interconnection function

$C(t,X) = C\,X$

where C is (6*6) constant matrix , and $U_T(t)$ is the total control signal:

$$U_T(t)= U^L(t)+U^g(t) \tag{15}$$

The global controller role is to minimize the effect of coupling between the different subsystems , which is ignored by the local controllers, i.e. it must minimize the effect of the interconnection function C(t,X)=CX . Therefore from (14) and (15) the global controller U^g can be obtained as follows :

$BU^g = - CX$

But since B is not a square matrix then using the pseudo inverse

$B^t B\, U^g = - B^t CX$

The global controller :

$$U^g= - (B^t B)^{-1}B^tCX \tag{16}$$

Using the local control (9) and the global control (16) then the total control (15) is obtained as :

$$U_T = -[R^{-1} B^t K + (B^t B)^{-1} B^t C]X + R^{-1} B^t h \tag{17}$$

by substituting (17) into (14) the closed-loop system is represented as follows

$$\dot{X} = [A_d - B_d R^{-1} B^t K - B_d (B^t B)^{-1} B^t C + C] X + B R^{-1} B^t h + E D \tag{18}$$

4.3 IMPLEMENTATION OF THE MULTILEVEL CONTROL

For the i^{th} subsystem

$$\dot{K}_i = -K_i A_i - A_i^t K_i + K_i B_i R_i^{-1} B_i t K_i - Q_i \qquad K_i (T) = 0$$

$$\dot{h}_i = (K_i B_i R_i^{-1} B_i^t - A_i^t) h_i + K_i E_i D_i \qquad h_i (T) = 0$$

Qi=diag[q1 q2]

Ri=r[I]

q_l, q_2 : weights for W_i and T_i respectively

r : weight for the control U_i

A_i , B_i , E_i : are the i^{th} subsystem matrices

K_i , h_i: feedback and feed forward gains of the subsystem respectively .

Optimal Local Controllers (First Level Controllers)

$U_i^{*L} = -R_i^{-1} B_i^t K_i X_i + R_i^{-1} B_i^t h_i$

Then for the three subsystems the local controllers :

$U_1^{*L} = 3.9/r[-K_{11}W_1 - K_{12}T_1 + h_1]$

$U_2^{*L} = 2.5/r[-K_{11}W_1 - K_{12}T_1 + h_1]$

$U_3^{*L} = 3.7/r[-K_{11}W_1 - K_{12}T_1 + h_1]]$

Where K_{ll}, K_{l2} and h_l are obtained for each subsystem as the solution of Riccati equation and using the proper A,B and E matrices .

Global Controllers (Second Level Control)

$U^g = -(B^t B)^{-1} B^t CX$

Then for the three subsystems the global controllers :

$U_1^g = 0$

$U_2^g = 1.2\ T_1$

$U_3^g = 0.2457\ T_1 + 1.1\ T_2$

Total Control

$U_T = U^{*L} + U^g$

Then for the three subsystems the total controllers :

$U_{1T}= 3.9/r[-K_{11}W_1-K_{12}T_1+h_1]$

$U_{2T}= 2.5/r[-K_{11}W_1-K_{12}T_1+h_1] + 1.2\ T_1$

$U_{3T}= 3.7/r[-K_{11}W_1-K_{12}T_1+h_1]+ 0.2457\ T_1 + 1.1\ T_2$

4.4 Algorithm of optimal multilevel control:

1.Given $K_{11}(T) = K_{12}(T) = K_{21}(T) = K_{22}(T) = 0$.

$h_1 (T) = h_2 (T) = 0$

for different r , q_1, q_2 , d_1 and d_2 solve Riccati equation (K_{11},K_{12},K_{21},K_{22},h_1 and h_2) in backward sequence in time using 4th order Runge-kutta method with time increment h=0.001 sec. and reserve K_{11}, K_{12}, K_{21}, K_{22}, h_1 and h_2 as a time sequence from T=t_f the final time until T=0.

2- Repeat step 1. For the three different subsystems and reserve K_{ij} and h_i gains for each subsystems (off- line) .
3- Given $W_1(0) = W_2(0) = W_3(0) = 0$ (Initial conditions)
$T_1(0) = T_2(0) = T_3(0) = 0$ (Initial conditions)
Solve the system closed-loop differential equations in forward sequence in time from T= 0 using the proper time sequence of feed back gains K_{ij} and feed forward gains h_i.
4- Use the updated values of the states and the proper data of K_{ij} and h_i and solve for the states until system reaches steady state.
5- Plot W_i and T_i versus time for i=1,2,3.

V. SUBOPTIMAL MULTILEVEL CONTROL

In designing the optimal local controllers for the three subsystems (first level) , the Kij and hi gains solved and reserved as a time sequence and used in the proper sequence when solving for the states (optimal design). Suboptimal local controllers for the three subsystems are designed such that the steady state values of the Kij and hi gains are used in solving for the system states. This will save the computer time and memory required since it is not needed to compute and reserve Kij and hi gains from T = t_f to T = 0 but until Kij and hi gains reach their steady state values , and reserve the steady state values only.

5.1 Algorithm of Suboptimal multilevel control:

1.Given $K_{11}(T) = K_{12}(T) = K_{21}(T) = K_{22}(T) = 0$.

$h_1 (T) = h_2 (T) = 0$

for different r , q_1, q_2 , d_1 and d_2 solve Riccati equation (K_{11},K_{12},K_{21},K_{22},h_1 and h_2) in backward sequence in time using 4th order Runge-kutta method with time increment h=0.001 sec. and reserve the steady state values of K_{11}, K_{12}, K_{21}, K_{22}, h_1 and h_2 from T= t_f the final time until it reaches its steady state values.

2- Repeat step 1. For the three different subsystems and reserve K_{ij} and h_i gains for each subsystems (off- line) .
3- Given $W_1(0) = W_2(0) = W_3(0) = 0$ (Initial conditions)
$T_1(0) = T_2(0) = T_3(0) = 0$ (Initial conditions)
Solve the system closed-loop differential equations in forward sequence in time from T= 0 using the proper steady state values of feed back gains K_{ij} and feedforward gains h_i.
4- Use the updated values of the states and the proper steady state values of K_{ij} and h_i and solve for the states until system reaches settles down.
5- Plot W_i and T_i versus time for i=1,2,3.

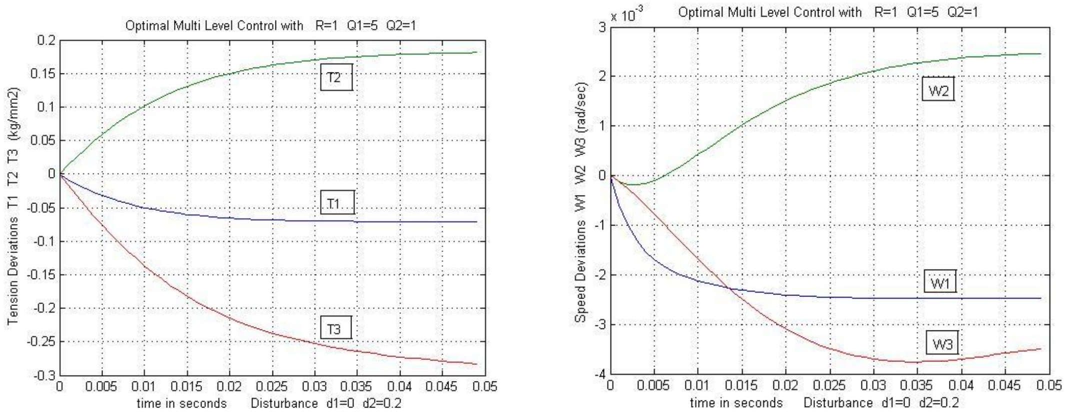

Figure 5 Optimal Multi Level Control d1=0 d2=0.2

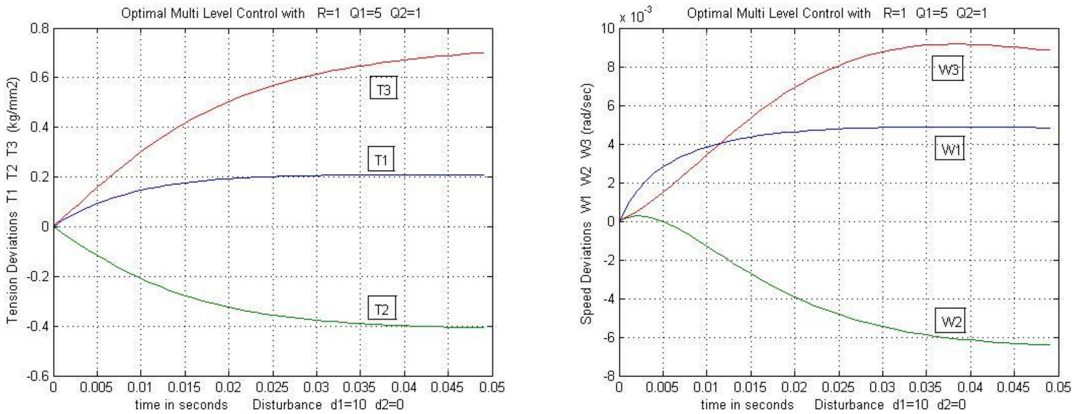

Figure 6 Optimal Multi Level Control d1=10 d2=0

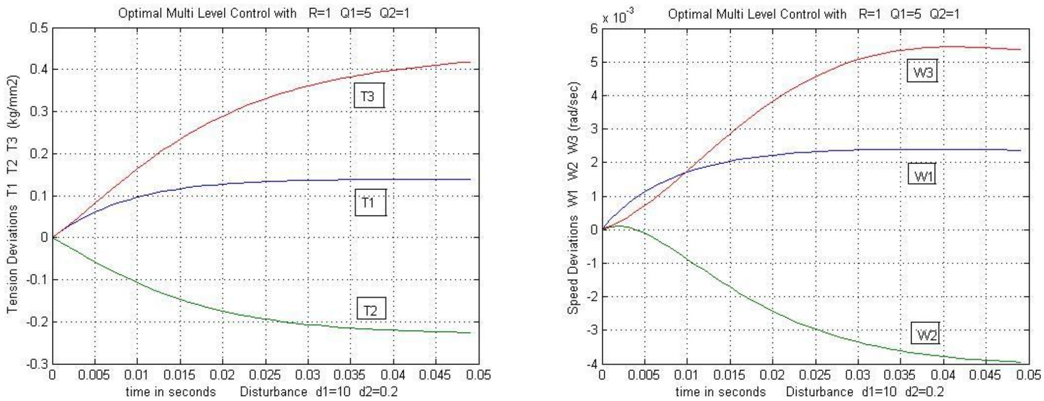

Figure 7 Optimal Multi Level Control d1=10 d2=0.2

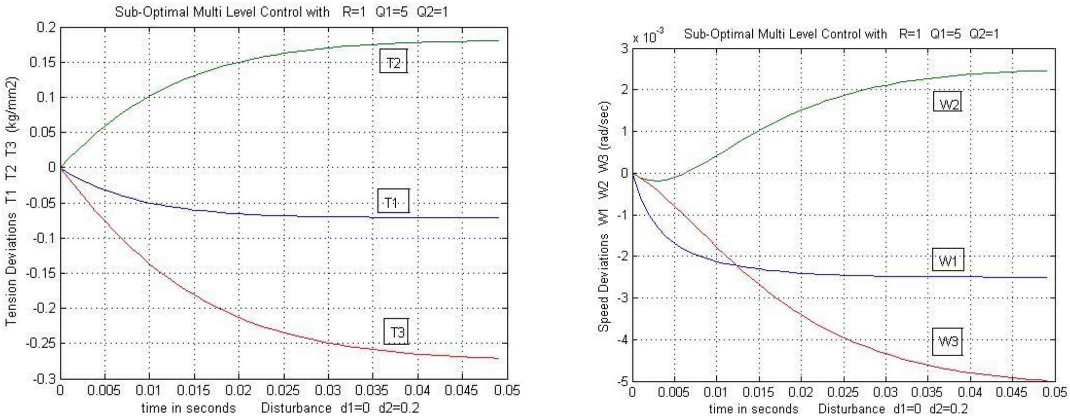

Figure 8 Sub-Optimal Multi Level Control d1=0 d2=0.2

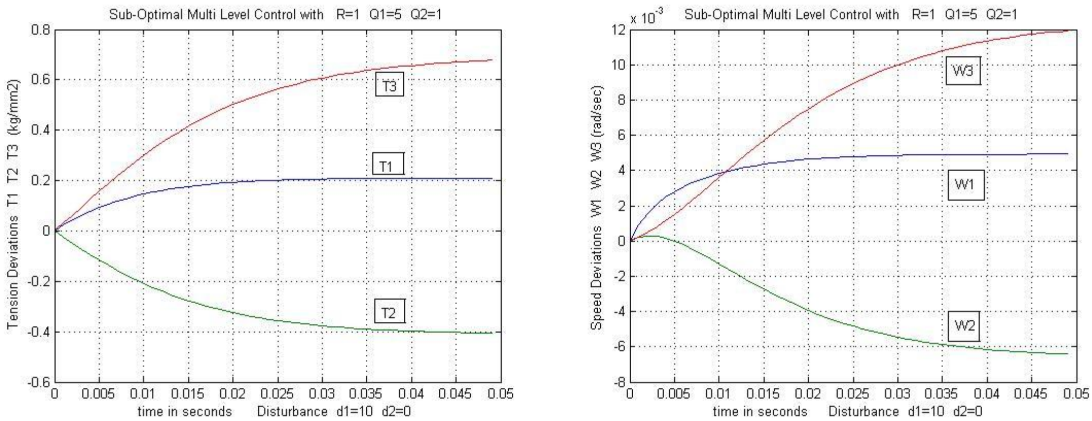

Figure 9 Sub-Optimal Multi Level Control d1=10 d2=0

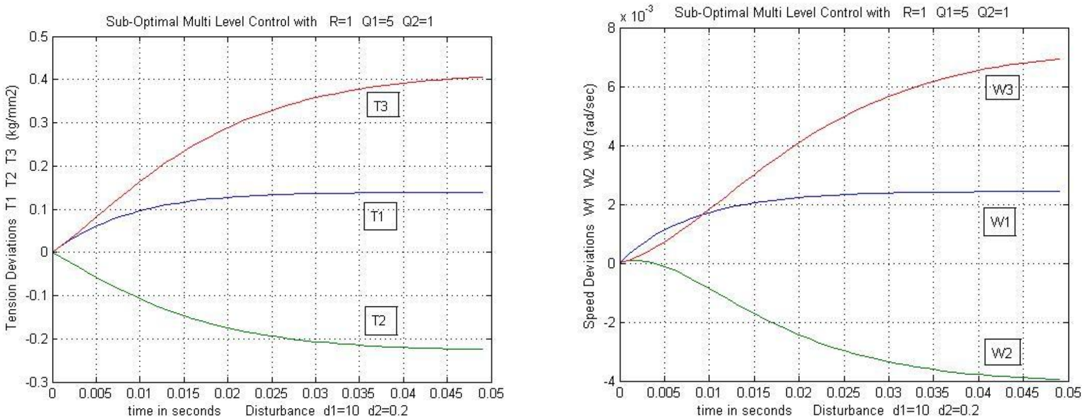

Figure 10 Sub-Optimal Multi Level Control d1=10 d2=0.2

VI. CONCLUSIONS

1. For this type of model, the controller usually has both feedback and feedforward components.
2. The disturbance d_1 and d_2 are affecting all system states but they are more effective in the tension states than the speed states .
3. System response due to disturbance only is obtained for the disturbances combinations $d_1=0$, $d_2=0.2$ in Figure 2 and for $d_l=10$, $d_2=0$ in Figure 3 and for $d_l=10$, $d_2=0.2$ in Figure 4 .The system under consideration is speed control therefore, speed control loops will be designed in all control techniques developed and applied to the finishing mill

system .

4 Optimal controllers are complex in implementation and maintenance and are highly sensitive to parameter variations.

5 A suboptimal controller which uses the steady state values of the K and h gains saves the computer time and memory required and shows, almost, compatible results with the optimal controller.

6 To modify the K and h gains of the control system to cope with certain operational requirement, an off- line solution should be carried out.

7 The global controller (coordination) U^g is not updating the first subsystems local controller, that's due to two points, the structure of the interconnection matrix C_i with the first row and the first column both are zeros reflecting the coupling between the different subsystems , and the control matrix for the first subsystems B_1 which reflects the fact that the system under consideration is speed control .

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